Roll No. ....

Total Pages : 3

BT-3/D-24

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MATHEMATICS–III

Paper : BS-205A

Time : Three Hours]

[Maximum Marks : 75

Note : Attempt any *five* questions.

1. (a) Discuss the convergence of the series

$$\sum_{n=1}^{\infty}\left(\frac{n!}{(n^n)^2}\right).$$

(b) Discuss the convergence or divergence of the series

$$\frac{x}{1.2}+\frac{x^2}{2.3}+\frac{x^3}{3.4}+..... \quad x > 0.$$

2. (a) Obtain the Fourier series for $f(x) = e^{-x}$ in the interval $0 < x < 2\pi$.

(b) Obtain the Fourier expansion of $x \sin x$ as a cosine series in the interval $(0, \pi)$ and deduce that

$$\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5.7}---=\frac{\pi-2}{4}.$$

3. (a) Solve $\frac{dy}{dx}=\frac{x^3+y^3}{xy^2}$ using exact differential equation.

(b) Solve the differential equation

$$\left(\frac{dy}{dx}\right)^2-5\frac{dy}{dx}+6=0.$$

4. (a) Solve $\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{3x}$.

(b) Solve by the method of variation of parameters :

$$\frac{d^2y}{dx^2} + y = \operatorname{cosec} x.$$

5. (a) Change the order of integration in the interval :

$$\int_0^1 \int_{x^2}^{2-x} xy\, dx\, dy.$$

(b) Show that area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$.

6. (a) Evaluate

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\, dx\, dy\, dz.$$

(b) Calculate by double integration, the volume generated by the revolution of the cardioid $r = a(1 - \cos\theta)$.

7. (a) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, show that grad $\frac{1}{r} = \frac{-\vec{r}}{r^3}$.

(b) Find the directional derivative of $\phi = x^2yz + 4xz^2$ at the point (1, −2, −1) in the direction of the vector $2\hat{i} - \hat{j} - 2\hat{k}$.

3. (a) Evaluate the line integral

$$\int_C (x^2 + xy)dx + (x^2 + y^2)dy,$$

where C is the square formed by the lines $x = \pm 1$, $y = \pm 1$.

(b) Using Green's Theorem, evaluate

$$\int_C (y - \sin x)dx + \cos x\, dy,$$

where C is the plane triangle enclosed by the lines

$$y = 0,\ x = \frac{\pi}{2} \text{ and } y = \frac{2}{\pi}x.$$
